

**Final Report of the UGC Minor Research Project  
MRP(S)-0657/13-14/KLKA004/UGC-SWRO**

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**UNIVERSITY GRANTS COMMISSION**

on

**Advanced study on Set- graceful and Set-Sequential  
Graphs**

by

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## Declaration

I ,Ragi Puthan Veettil,here by declare that this report is an original work done by me under UGC Minor Research Project MRP(S)-0657/13-14/KLKA004/UGC-SWRO dated 28-03-14 in the P G Department of Mathematics,PRNSS College ,MATTANUR and has not been submitted by me elsewhere for the award of any degree,diploma,title or recognitiion,before.

Mattanur

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July 15, 2017

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# Chapter 1

## INTRODUCTION

Graph Theory came into existence during the first half of the 18th century when Euler settled a famous unsolved problem of his day called the Königsberg Bridge Problem. There were two islands linked to each other and to the banks of the Pregel River by seven bridges. The problem was to begin at any of the four land areas, walk across each bridge exactly once and return to the starting point. In proving that the problem is unsolvable, Euler replaced each land area by a point and each bridge by a line joining the corresponding points there by producing a graph.

This area didn't start to develop into an organised branch of Mathematics until the second half of the 19th century, and there was not even a book on the subject until the first half of the 20th century. Since the second half of the 20th century, the subject has exploded.

In this work I studied set valuation of the graph in detail. In the first chapter definition and some preliminary ideas are given.

In the second chapter some classes of Set graceful is studied. Necessary condition for certain operations on graphs to be Set graceful is also done.

In the third chapter a Graphical realisation of Ideal is defined and its some properties were studied.

Some open problems were given for further research .

For all the notations used in this project we use Frank Hararay.  
All the graphs in this paper are simple and finite.

# Chapter 2

## PRELIMINARIES

In this chapter we give definition and some basic properties of graphs necessary for this project.

### 2.1 Graph

A graph  $G$  consists of a finite nonempty set  $V = V(G)$  of  $p$  vertices together with a prescribed set  $E$  of  $q$  unordered pairs of distinct vertices of  $V$ . Each pair  $e = (u, v)$  of vertices in  $E$  is an edge of  $G$ , and  $e$  is said to join  $u$  and  $v$ . We write  $e = uv$  and say that  $u$  and  $v$  are adjacent vertices; vertex  $u$  and edge  $e$  are incident with each other, as are  $v$  and  $e$ . If two distinct edges  $e_1$  and  $e_2$  are incident with a common vertex, then they are adjacent edges. A graph with  $p$  vertices and  $q$  edges is called a  $(p, q)$  graph.

Two edges are said to be parallel if their end vertices are same.

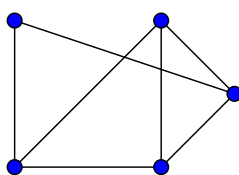


Figure 2.1: Graph

#### 2.1.1 Labeled Graph

A graph  $G$  is labeled when the  $p$  vertices are distinguished from one another by names such as  $v_1, v_2, \dots, v_p$ .

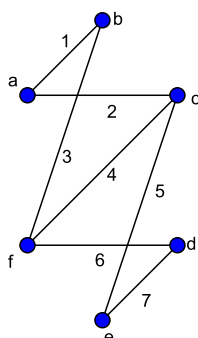


Figure 2.2: Labeled Graph

### 2.1.2 Simple Graph

A graph which has no loops and no parallel edges is called a simple graph.

### 2.1.3 Complete Graph

A graph is said to be complete if every pair of distinct vertices is joined by an edge. A complete graph on  $n$  vertices is denoted by  $K_n$ .  $K_n$  has  $\frac{n(n-1)}{2}$  edges.

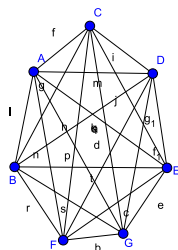


Figure 2.3: The complete graph on seven vertices

## 2.2 Walks and Connectedness

A walk of a graph  $G$  is an alternating sequence of vertices and edges  $v_0e_1v_1e_2v_2e_3v_3\dots e_nv_n$  in which each edge is incident with the two vertices immediately preceding and following it. It is called a  $v_0 - v_n$  walk.

If  $v_0 = v_n$  it is called a closed walk and is open otherwise.

A walk is a trail if all the edges are distinct, and a path if all the vertices are distinct. A path with  $n$  vertices is denoted by  $p_n$ .

If the walk is closed, then it is a cycle provided its  $n$  vertices are distinct and  $n \geq 3$ . A cycle with  $n$  vertices is denoted by  $C_n$ .

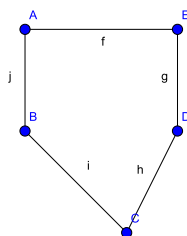


Figure 2.4: A cycle on 5 vertices

A graph is connected if every pair of vertices are joined by a path, otherwise it is called a disconnected graph.

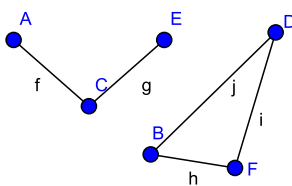


Figure 2.5: A disconnected graph



# Chapter 3

## Some Classes of Set Graceful Graphs

### 3.1 Introduction

Motivated from number valuations of graphs, B.D. Acharya defined Set indexer, set graceful and set sequential graphs in 1983. Subsequently many researchers have worked with set valued graph and found some classes of these graph. Many problems in this area are to be solved.

In this chapter we have studied some binary operations on certain classes of set graceful graphs. Necessary conditions for certain graphs to be set graceful and biset graceful have been studied.

### 3.2 Preliminaries

**Definition 3.2.1.** [2]

Let  $G = (V, E)$  be a graph,  $X$  be non empty set and  $2^X$  denote the set of all subsets of  $X$ . A set indexer of  $G$  is an injective set valued function  $f : V(G) \rightarrow 2^X$  such that the function  $f^\oplus : E(G) \rightarrow 2^X - \{\emptyset\}$  defined by  $f^\oplus(UV) = f(U) \oplus f(V)$  for every  $U, V \in E(G)$  is also injective, where  $\oplus$  denotes the symmetric difference of sets.

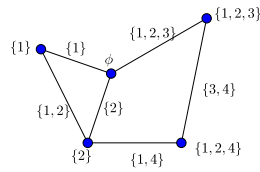
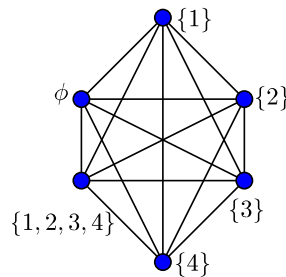


Figure 3.1: A set indexed graph.

**Theorem 3.2.2.** [2] *Every graph has a set indexer.*

**Definition 3.2.3.** [2]

A graph  $G = (V, E)$  is said to be set graceful if there exist a nonempty set  $X$  and a set indexer  $f : V(G) \rightarrow 2^X$  such that  $f^\oplus(E(G)) = 2^X - \{\emptyset\}$ , such an indexer being called set graceful labeling of  $G$ .

Figure 3.2: set graceful labeling of  $K_6$ .

The following theorem gives a straight forward necessary condition for a graph  $G$  to be set graceful.

**Theorem 3.2.4.** [2] *If  $G = (p, q)$  be a set graceful graph, then  $q = 2^m - 1$  and  $p \leq q + 1$ .*

*The smallest such positive integer  $m$  is called the set indexing number.*

**Definition 3.2.5.** [9] *Let  $G_1$  and  $G_2$  be two graphs with disjoint vertex sets  $V_1$  and  $V_2$  then, their join  $G_1 + G_2$  consists of  $G_1 \cup G_2$  and all the edges joining  $V_1$  with  $V_2$ .*

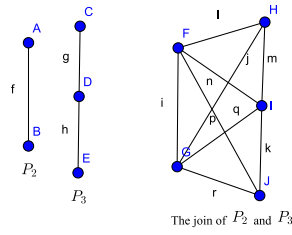


Figure 3.3: The join of  $P_2$  and  $P_3$

**Definition 3.2.6.** [9] Let  $G_1 = (V_1, E_1)$  and  $G_2 = (V_2, E_2)$  are two graphs. Then their cross product  $G_1 \times G_2$  is defined as follows, two points  $u = (u_1, u_2)$  and  $v = (v_1, v_2)$  in  $V = V_1 \times V_2$  are adjacent in  $G_1 \times G_2$  if  $u_1 = v_1$  and  $u_2$  adjacent to  $v_2$  or  $u_2 = v_2$  and  $u_1$  adjacent to  $v_1$ .

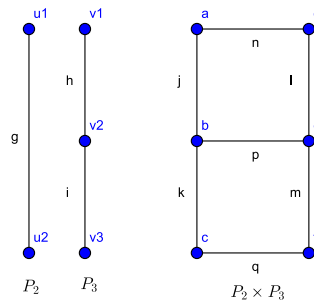
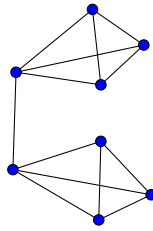


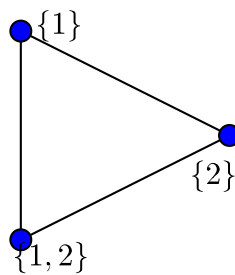
Figure 3.4: The cross product of  $P_2$  and  $P_3$

**Definition 3.2.7.** [6] The corona of two graphs  $G_1 = (p_1, q_1)$  and  $G_2 = (p_2, q_2)$  is the graph  $G_1 \circ G_2$  formed from one copy of  $G_1$  and  $|V(G_1)|$  copies of  $G_2$  where the  $i^{th}$  vertex of  $G_1$  is adjacent to every vertex in the  $i^{th}$  copy of  $G_2$ .

Figure 3.5: The corona of  $K_2$  and  $K_3$ 

**Theorem 3.2.8.** [6]

The complete graph  $K_n$  is set graceful if and only if  $n \in \{1, 2, 3, 6\}$ .

Figure 3.6: Set Graceful Labeling of  $K_3$ 

**Definition 3.2.9.** [9] The line graph of a graph  $G$ , denoted by  $L(G)$ , has  $E(G)$  as its vertex set with two vertices of  $L(G)$  are adjacent whenever the corresponding edges of  $G$  are adjacent.

### 3.3 Some binary operations on set graceful graphs.

**Theorem 3.3.1.** A necessary condition for the cross product  $G_1 \times G_2$  of two set graceful graphs  $G_1 = (p_1, q_1)$  and  $G_2 = (p_2, q_2)$  to be set graceful is that  $p_1$  and  $p_2$  are of opposite parities.

*Proof.* Let  $G_i$  has  $p_i$  vertices and  $q_i$  edges for  $i = 1, 2$ . Then  $q_i = 2^{n_i} - 1$  where  $n_i$  are the set indexing number for each  $G_i$ . If  $G_1 \times G_2$  is set graceful then

$p_1(2^{n_2}-1)+p_2(2^{n_1}-1) = 2^k-1$  for some  $k \in N$ . Then  $p_12^{n_2}+p_22^{n_1} = 2^k-1+p_1+p_2$ . Hence  $p_1 + p_2$  must be odd. Therefore either  $p_1$  is odd and  $p_2$  is even or  $p_2$  is odd and  $p_1$  is even. Hence the theorem.  $\square$

**Remark 3.3.2.** *Converse of the above result is not true as  $K_3 \times K_2$  is not set graceful.*

**Theorem 3.3.3.** *If  $T$  is a Set graceful tree with  $n$  vertices, then  $K_1 + T$  is Set graceful.*

*Proof.* Let  $G = K_1 + T$ . Since  $T$  is Set Graceful,  $T$  has  $2^n$  vertices and  $2^n - 1$  edges for some  $n \in N$ . Take  $V(T) = \{v_1, v_2, \dots, v_{2^n}\}$  and  $V(K_1) = \{u\}$  so that  $V(G) = \{u, v_1, v_2, \dots, v_{2^n}\}$ . Then  $|V(G)| = 2^n + 1$  and  $|E(G)| = 2^{n+1} - 1$ . Define  $X_n = \{1, 2, 3, \dots, n\}$ . Define the labelling  $f : V(G) \rightarrow 2^{X_{n+1}}$  as follows. Label  $u$  by  $X_{n+1}$  and  $v_i$  by subsets of  $X_n$ , for each  $i = 1, 2, \dots, 2^n$  in such away that  $T$  is Set graceful. Then the labelling of edges of  $G$  are injective and  $f^\oplus(G) = 2^{X_{n+1}} - \{\emptyset\}$ . Therefore  $G$  is Set graceful.  $\square$

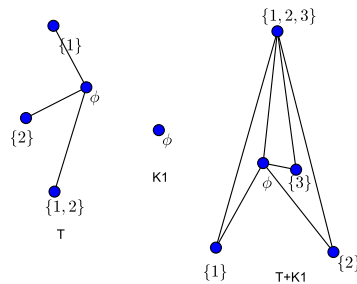


Figure 3.7: Set Graceful Labeling of join of  $K_1$  and a tree

**Theorem 3.3.4.** *A necessary condition for the corona  $G_1 \circ G_2$  of two set graceful graphs  $G_1 = (p_1, q_1)$  and  $G_2 = (p_2, q_2)$  to be set graceful is that  $p_1 \equiv 0 \pmod{2}$  or  $p_2 \equiv 1 \pmod{2}$  or both.*

*Proof.* Let  $G_i$  has  $p_i$  vertices and  $q_i$  edges for  $i = 1, 2$ . Then  $q_i = 2^{n_i} - 1$  where  $n_i$  are the set indexing number for each  $G_i$ . Then  $G_1 \circ G_2$  has  $p_1(1 + p_2)$  vertices and  $q_1 + p_1(p_2 + q_2)$  edges.

Then if  $G_1 \circ G_2$  is set graceful we have  $q_1 + p_1(p_2 + q_2) = 2^k - 1$  for some  $k \in N$ . Therefore  $2^{n_1} - 1 + p_1 p_2 + p_1(2^{n_2} - 1) = 2^k - 1$ .

Hence  $2^{n_1} + p_1 2^{n_2} - 2^k = p_1 - p_1 p_2$ .

That is 2 divides  $p_1(1 - p_2)$ .

Hence  $p_1 \equiv 0 \pmod{2}$  or  $p_2 \equiv 1 \pmod{2}$  or both.  $\square$

**Remark 3.3.5.** *Converse of the above result is not true as  $K_3 \circ K_2$  is not set graceful.*

**Definition 3.3.6.** [8] *The Boolean lattice  $BL_n$  is the graph whose vertex set is the set of all subsets of  $\{1, 2, 3, \dots, n\}$  where two subsets  $X$  and  $Y$  are adjacent if their symmetric difference has precisely one element.*

**Observation 3.3.7.** *No Boolean lattice is set graceful as labeling of edges is not injective.*

**Definition 3.3.8.** [6] *An  $n$ -sun is a graph that consists of a cycle  $C_n$  and an edge terminating in vertex of degree one attached to each vertex of  $C_n$ .*

**Observation 3.3.9.** *The  $n$ -sun is not set graceful for every  $n$ .*

## 3.4 Biset Graceful Graphs

**Definition 3.4.1.** [6] *A graph  $G$  is called biset graceful if  $G$  and its line graph  $L(G)$  are set graceful.*

**Theorem 3.4.2.** *If  $G = (p, q)$  is a biset graceful graph, then sum of squares of degrees of vertex is divisible by 4, when  $p \geq 3$ .*

*Proof.* Suppose  $G = (p, q)$  be a biset graceful graph. Then  $q = 2^m - 1$  for some  $m \in N$ . Also  $L(G) = (-q, -q + (\sum d_i^2)/2)$ . Then  $-q + (\sum d_i^2)/2 = 2^n - 1$  for some  $n \in N$ .

But then,  $-2^m + 1 + (\sum d_i^2)/2 = 2^n - 1$

$\sum d_i^2 = 4(2^{n-1} + 2^{m-1} - 1)$ .

Hence the result.  $\square$

**Remark 3.4.3.** *When  $p = 2$ ,  $\sum d_i^2$  is divisible by 2.*

**Remark 3.4.4.**  $P_2$  is the only biset graceful path. Since no path  $P_n, n \geq 3$  is set graceful and both  $P_2$  and  $L(P_2) = K_1$  are set graceful.

**Problem 3.4.5.**  $K_3 \times K_6$  is set graceful

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# Chapter 4

## Graphical Realisation of Ideals

### 4.1 Introduction

Motivated from number valuations of graphs, B.D. Acharya defined Set indexer, set graceful and set sequential graphs in 1983. Let  $G = (V, E)$  be a graph,  $X$  be non empty set and  $2^X$  denote the set of all subsets of  $X$ . A set indexer of  $G$  is an injective set valued function  $f : V(G) \rightarrow 2^X$  such that the function  $f^\oplus : E(G) \rightarrow 2^X - \{\emptyset\}$  defined by  $f^\oplus(UV) = f(U) \oplus f(V)$  for every  $U, V \in E(G)$  is also injective, where  $\oplus$  denotes the symmetric difference of sets. A graph  $G = (V, E)$  is said to be set graceful if there exist a nonempty set  $X$  and a set indexer  $f : V(G) \rightarrow 2^X$  such that  $f^\oplus(E(G)) = 2^X - \{\emptyset\}$ , such an indexer being called set graceful labeling of  $G$ . A *filter* on a non-empty set  $X$  is a non-empty family  $\mathcal{F}$  of subsets of  $X$  which satisfies the following conditions.

- (i)  $\emptyset \notin \mathcal{F}$
- (ii)  $\mathcal{F}$  is closed under finite intersections
- (iii) if  $B \in \mathcal{F}$  and  $B \subset A$ , then  $A \in \mathcal{F}$  for all  $A, B \subset X$ .

**Definition 4.1.1.** [6] A filter  $\mathcal{F}$  on a set  $X$  is said to be graphically realisable if there exists a connected graph  $G = (V, E)$  and a set indexer  $f : V \rightarrow 2^X$  such that  $f(V) = \mathcal{F}$ . The graph  $G$  is said to be the graphical realisation of the filter  $\mathcal{F}$ .

### 4.2 Ideals and Their Graphical Realisations

**Definition 4.2.1.** An *ideal* in a non-empty finite set  $X$  is a non-empty family  $\mathcal{I}$  of subsets of  $X$  which satisfies the following conditions.

- (ii)  $\mathcal{I}$  is closed under finite unions.
- (iii) if  $B \in \mathcal{I}$  and  $A \subset B$ , then  $A \in \mathcal{I}$  for all  $A, B \subset X$ .

An ideal  $\mathcal{I}$  is the family of subsets of a set  $A$  in  $\mathcal{I}$  with maximum cardinality. If  $A$  is of cardinality  $k$  then  $\mathcal{I}$  is of cardinality  $2^k$ . An ideal containing  $X$  is the power set  $2^X$  of  $X$ . The ideal which does not contain  $X$  is called a proper ideal.

Motivated from the graphical realisation of filters, here we introduce the concept of graphical realisation of ideals for a given non-empty finite set  $X$ .

**Definition 4.2.2.** *An ideal  $\mathcal{I}$  on a set  $X$  is said to be graphically realisable if there exists a connected graph  $G = (V, E)$  and a set-indexer  $f : V \rightarrow 2^X$  such that  $f(V) = \mathcal{I}$ .*

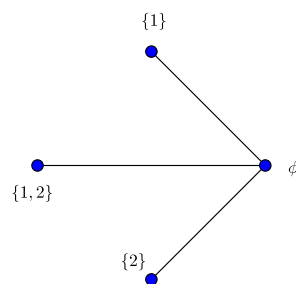


Figure 4.1:  $K_{1,3}$  is the graphical realisation of the ideal  $\{\phi, \{1\}, \{2\}, \{1,2\}\}$

**Theorem 4.2.3.** *Every ideal  $\mathcal{I}$  in a given non-empty finite set  $X$  is graphically realisable.*

*Proof.* Suppose that  $\mathcal{I}$  is an ideal in a nonempty finite set  $X$ . If  $A$  is the element in  $\mathcal{I}$  with maximum cardinality  $n$  then all other elements in  $\mathcal{I}$  are subsets of  $A$ . Hence  $\mathcal{I}$  has  $2^n$  elements. Assign  $2^n$  elements of  $\mathcal{I}$  to  $2^n$  vertices and their symmetric differences to the  $2^n - 1$  edges of  $K_{1,2^n-1}$ , the star graph, respectively. Then the edge set are exactly all nonempty subsets of  $A$ . Hence every ideal  $\mathcal{I}$  is graphically realisable  $\square$

From the proof of above theorem we can conclude that :

**Corollary 4.2.4.** *The star graph  $K_{1,2^n-1}$  is set graceful with respect to a set  $X$  of cardinality  $n$ .*

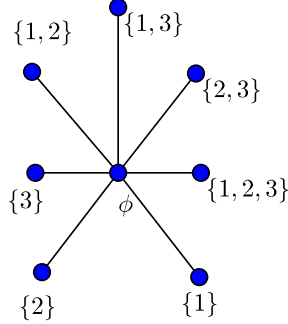


Figure 4.2:  $K_{1,7}$  is set graceful.

The following result gives a connection between graphical realisation of an ideal and that of a filter.

**Theorem 4.2.5.** *A graph  $G = (V, E)$  is a graphical realisation of a proper ideal if and only if it is a graphical realisation of a filter.*

*Proof.* Suppose  $G = (V, E)$  is a graphical realisation of an ideal  $\mathcal{I}$  in a set  $X$ . Let  $\mathcal{F}$  be the family of all subsets of  $X$  which are complement to each of the elements of  $\mathcal{I}$ . We claim that  $\mathcal{F}$  is a filter in  $X$ .

- (i)  $\emptyset \notin \mathcal{F}$ , since  $\mathcal{I}$  is a proper ideal.
- (ii) If  $A, B \in \mathcal{F}$ , then  $A^c$  and  $B^c$  are in  $\mathcal{I}$ . Therefore their union  $A^c \cup B^c \in \mathcal{I}$  since  $\mathcal{I}$  is an ideal. That is  $(A \cap B)^c \in \mathcal{I}$ . Therefore  $A \cap B \in \mathcal{F}$ .
- (iii) Let  $A \subset B$  and  $A \in \mathcal{F}$ . Then  $B^c \subset A^c$  and since  $A^c \in \mathcal{I}$  we have  $B^c \in \mathcal{I}$ . Therefore  $B \in \mathcal{F}$ .

Hence  $\mathcal{F}$  is a filter in  $X$ .

Now  $|\mathcal{I}| =$  number of subsets of  $A$ , where  $A$  is an element of  $\mathcal{I}$  with maximum cardinality.

$|\mathcal{F}| =$  number of supersets of  $A^c$  in  $X = 2^{|X| - (|X| - k)} = 2^k = |\mathcal{I}|$ . Then for the graph  $G' = (V', E')$  obtained from  $G$  by assigning the vertices of  $G$  to its complement in  $X$  we have  $E' = E$  since  $A \oplus B = A^c \oplus B^c$ . Then  $G'$  is a graphical realisation of the filter  $\mathcal{F}$ . and  $G$  is isomorphic to  $G'$ .

Conversely assume that  $G' = (V', E')$  is a graphical realisation of a filter  $\mathcal{F}$  in  $X$ . Then the graph  $G = (V, E)$  where  $E = E'$  and  $V$  is the set of all elements  $A^c$  where  $A \in V'$  is a graphical realisation of ideal in  $X$ .  $\square$

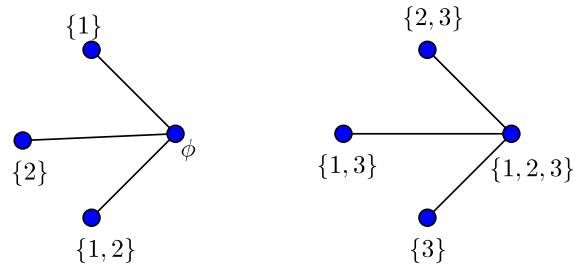


Figure 4.3:  $K_{1,3}$  as graphical realisation of an ideal and that of a filter

**Remark 4.2.6.** *The filter in the above theorem is called the dual filter of the ideal  $\mathcal{I}$ .*

**Theorem 4.2.7.** *[6] The graphical realisation of any filter is a tree.*

From the above theorem and theorem 2.5 we can conclude that :

**Corollary 4.2.8.** *The graphical realisation of any ideal is a tree.*

**Corollary 4.2.9.**  *$K_2$  is the only complete graph ,which is a graphical realisation of an ideal .*

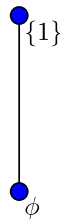


Figure 4.4:  $K_2$  is the graphical realisation of the ideal  $\{\{1\}, \phi\}$

**Corollary 4.2.10.** *The graphical realisation of an ideal of cardinality 1 is  $K_1$  and that of cardinality 2 is  $K_2$*

**Corollary 4.2.11.** *[6] The graphical realisation of a filter of cardinality  $2^2$  is isomorphic to  $K_{1,3}$ .*

From the above corollary and theorem 2.5 we can conclude that :

**Corollary 4.2.12.** *The graphical realisation of an ideal of cardinality  $2^2$  is isomorphic to  $K_{1,3}$*

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## OPEN PROBLEMS

1. Prove or disprove :  $K_3 \times K_6$  is set graceful.
2. For  $n \geq 2$  ,no path  $P_2^n$  can be a realisation of an ideal defined on a nonempty finite set  $X$ .
3. Find the non isomorphic graphical realisations of ideals of cardinality  $2^n$  for  $n \geq 3$ .
4. Determine the conditions required for a tree  $T$  to be a graphical realisation of a given ideal on  $X$ .

## Paper Presented

**Graphical Realisation of Ideals**-Presented in the three days MESMAC International Conference at MES Mampad College during February 14-16,2017.

## Paper Published

1. The presented paper **Graphical Realisation of Ideals** is in the process of publishing in the proceedings , **Singularities** ISSN-2348-3369 of the international conference.
2. The paper **Some classes of Set Graceful Graphs** is published in the Aryabhatta Journal of Mathematics and Informatics ,ISSN 0975-7139 (P), 2394-9309(E),Vol.09-Issue -01,(January-June-2017),743-749

## Seminars Participated

1. Three days workshop on **Functional Analysis at Central University Of Kerala** ,during 20-22 march 2014.
2. Two day national seminar on **Discrete Mathematics And Its Applications at KMM Government Women's College,Kannur** on 20th and 21st october 2014.
3. Short term refresher course for college teachers in Kerala state on **Algebra at KSOM,Calicut** during 08-11 January,2015 .
4. Short term refresher course for college teachers in Kerala state on **Topologies,Filters , &Uniformities.** at **KSOM,Calicut** during 12-15 November,2015.
5. Two day national workshop on **Number Theory & Works of Srinivasa Ramanujan** at **University of Mysore** on February 26 & 27,2016.